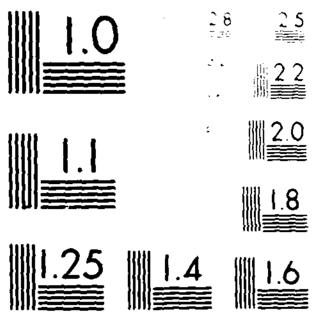
AD-A116 211 WISCONSIN UNIV-MADISON MATHEMATICS RESEARCH CENTER F/G 12/1
ESTIMATION OF VARIANCE OF THE RATIO ESTIMATOR: AN EMPIRICAL STU--ETC(U)
MAY 82 C WU, L DENG DAAG29-80-C-0041
UNCLASSIFIED MRC-TSR-2378 NL

1 OF 1

AD A

END
DATE
FILMED
07-82
DTIC



MICROCOPY RESOLUTION TEST CHART

AD A116211

MRC Technical Summary Report #2378

ESTIMATION OF VARIANCE OF THE RATIO ESTIMATOR: AN EMPIRICAL STUDY

Chien-Fu Wu and Lih-Yuan Deng

Mathematics Research Center
University of Wisconsin—Madison
610 Walnut Street
Madison, Wisconsin 53706

May 1982

(Received April 28, 1982)

DTIC FILE COPY

Approved for public release
Distribution unlimited

JUN 29 1982
A

Sponsored by

U. S. Army Research Office
P. O. Box 12211
Research Triangle Park
North Carolina 27709

National Science Foundation
Washington, DC 20550

82 06 29 050

UNIVERSITY OF WISCONSIN-MADISON
MATHEMATICS RESEARCH CENTER

ESTIMATION OF VARIANCE OF THE RATIO ESTIMATOR: AN EMPIRICAL STUDY

Chien-Fu Wu and Lih-Yuan Deng*

Technical Summary Report #2378
May 1982

ABSTRACT

Several estimators of variance of the ratio estimator in simple random sampling have been proposed in Cochran (1977), Fuller (1981), Royall and Cumberland (1978), Royall and Eberhardt (1975), Wu (1982). Their performances are compared on nine populations that reflect different features of natural populations encountered in practice. One criterion is the mean square error of the variance estimator as a point estimator of the variance of ratio; the other is the reliability of the associated t-interval. It turns out that the two criteria are not consistent. The apparent contradiction is resolved by a conditioning argument on an ancillary statistic, i.e., the reliability of the t-interval can be predicted by the closeness of the corresponding variance estimator to the conditional MSE of the ratio estimator on the ancillary statistic. Based on the empirical study, the jackknife estimator v_J and the estimator v_2 (and other asymptotically equivalent ones) are recommended. The good performance of these estimators is attributed to their ability in "capturing" the ancillary statistic.

AMS (MOS) Subject Classifications: 62D05, 62E25

Key Words: Ratio estimator, variance estimator, jackknife, regression estimator, ancillary statistic, conditional inference, superpopulation model, Monte Carlo

Work Unit Number 4 (Statistics and Probability)

*Department of Statistics, University of Wisconsin-Madison, 1210 West Dayton Street, Madison, WI 53706.

Sponsored by the United States Army under Contract No. DAAG29-80-C-0041. This material is based upon work supported by the National Science Foundation under Grant No. MCS-7901846.

SIGNIFICANCE AND EXPLANATION

The ratio estimator is an important estimation method in sample surveys when an auxiliary variable is available. There are many ways of estimating its variance. Their empirical performances are compared on nine populations according to two criteria. The more interesting one is the reliability of the confidence interval based on the t-statistic (ratio estimator-population mean)/estimated standard error. The jackknife variance estimator and other (asymptotically equivalent) estimators are recommended for practical purpose. Their good performance seems to be related to inference conditional on an appropriate ancillary statistic.

DTIC
COPY
INSPECTED
2

6

Dist
A

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the authors of this report.

ESTIMATION OF VARIANCE OF THE RATIO ESTIMATOR: AN EMPIRICAL STUDY

Chien-Fu Wu and Lih-Yuan Deng*

1. INTRODUCTION

This paper concerns the estimation of variance of the ratio estimator under simple random sampling. While the setting is simple, we hope this study will eventually lead to better understanding of the important problem of variance estimation in complex surveys. In fact even in this simple setting, the problem of choosing "good" variance estimators is unsettled. More than a dozen estimators, proposed in a span of some thirty years, are listed in Rao (1969) and Royall and Cumberland (1981). The majority of estimators are design-based, i.e., their justification and choice are based on the performance according to the probability mechanism that generates the sample. A few others, proposed by Royall and his collaborators, are model-based. According to this approach, the inference should be made conditional on the observed sample and a hypothetical superpopulation model. The sampling design becomes irrelevant. Other estimators, e.g. the jackknife, may not be justified exclusively by either approach.

Previous work on the comparison of variance estimators for ratio include, among others, Rao and Beegle (1967), Rao (1968, 1969), Rao and Rao (1971), Rao and Kuzik (1974), Royall and Eberhardt (1975), Royall and Cumberland (1978, 1981), Krewski and Chakrabarty (1981), and Wu (1982). The theoretical comparison of various variance estimators is made by assuming that the x and y populations satisfy some linear regression models (superpopulations). Although the results are sometimes exact, dependence on the

---

This paper will appear in the "Proceedings of the Conference on Scientific Inference, Data Analysis, and Robustness" (ed. by G.E.P. Box, et al.), Academic Press, New York, 1983.

---

*Department of Statistics, University of Wisconsin-Madison, 1210 West Dayton Street, Madison, WI 53706

Sponsored by the United States Army under Contract No. DAAG29-80-C-0041. This material is based upon work supported by the National Science Foundation under Grant No. MCS-7901846.

superpopulation parameters can be delicate. This prompted Royall and Eberhardt (1975) to study the model-robustness of some variance estimators. Their definition of robustness is restricted to the bias behavior of the variance estimators when the true parameters deviate from the assumed ones. Wu (1982) gave the first model-free comparison of some variance estimators by expanding the estimators and working on the leading terms of the expansion. Such a comparison is large sample in nature. On the empirical side, comparison is conducted on either natural populations or artificial populations simulated according to some superpopulation models. The bias and/or mean square error of the variance estimators are noted. Motivated by the prediction theory approach, Royall and Cumberland (1981) took a different approach by studying the conditional behavior of the variance estimators as a function of the x-sample mean $\bar{x}$. They showed that some variance estimators can behave drastically different over a range of the $\bar{x}$ values. They further argue that the conditional (on $\bar{x}$) mean of a variance estimator should closely follow the conditional (on $\bar{x}$) mean square error of the ratio estimator. We will come back to this point in §5 and §6.

The work to be presented is empirical and is in part inspired by the very stimulating paper of Royall and Cumberland (1981). Their approach to empirical study can be further improved in three respects. They did not consider estimator v_2 defined in §2, which is motivated by the probability sampling theory and is popular in practice. In any effort to criticize the more traditional sampling theory approach, it seems fair to consider both v_0 and v_2. See also J. N. K. Rao's discussion of Royall and Cumberland (1978). It will be shown later that v_2 is better than v_0 in several desirable respects. Their conclusion in favor of the prediction theory approach could have been more convincing had they included the stronger "rival" v_2 in their study. To remedy this we have included several additional estimators in our study. The six natural populations they chose look artificial in that they are all well fitted by model (2), i.e. straight lines through the origin with increasing residuals. We consider nine populations, six identical to theirs and three incorporating violations of three key assumptions of the linear regression model (2) that typically underlines the use of ratio estimator. Detail is in §4. Besides studying the conditional behavior of the variance estimators in "tracking" the conditional MSE of the ratio estimator, an innovation due to Royall and Cumberland, we also study the bias and MSE of the variance estimators as estimators of the unconditional MSE of the ratio estimator and, more importantly, the actual coverage probabilities of the associated interval estimates of the y-population mean $\bar{Y}$ as compared with the nominal ones.

2. RATIO ESTIMATOR AND ITS VARIANCE

Suppose that a population consists of N distinct units with values (y_i, x_i), where $x_i > 0$ for $1 \leq i \leq N$. A simple random sample of size n is taken without replacement from the population. Denote the sample and population means of y_i and x_i by $\bar{y}, \bar{x}$ and $\bar{Y}, \bar{X}$ respectively. The ratio estimator

$$\hat{\bar{y}}_R = \bar{y} \frac{\bar{X}}{\bar{x}} \tag{1}$$

is a popular estimator of $\bar{Y}$. It is simple to use in practice. It combines efficiently the covariate information in x_i when y_i and x_i are roughly positively correlated. It is the best linear unbiased predictor of $\bar{Y}$ under the following superpopulation model (Brewer, 1963; Royall, 1970)

$$y_i = \beta x_i + \varepsilon_i , \tag{2}$$

where ε_i are independent with mean zero and variance $\sigma^2 x_i$. The ratio estimator possesses other desirable properties. For example, it is robust against extreme values in the individual ratios y_i/x_i (Rao, 1978). Traditionally the ratio estimator is favored over the regression estimator mainly for computational ease in handling large data sets. Given the present capacity of computers this should be less of a concern. Fuller (1977) gave examples to show that ratio estimation can be much less efficient than regression estimation. We believe it is time that more attention should be given to regression estimation.

There is no closed form for MSE $\hat{\bar{y}}_R$ or Var $\hat{\bar{y}}_R$. Both can be approximated by the approximate variance (Cochran, 1977, p. 155)

$$V_{appr} = \frac{1-f}{n}\frac{1}{N-1}\sum_1^N \left(y_i - \frac{\bar{Y}}{\bar{X}} x_i\right)^2 , \tag{3}$$

where $f = n/N$ is the sampling fraction. For large samples the approximation is adequate. But for small sample size ($n < 12$) V_{appr} can seriously underestimate MSE (Rao, 1968 or Cochran, 1977, p. 164). The most standard estimator of V_{appr} is its sample analogue

$$v_0 = \frac{1-f}{n}\frac{1}{n-1}\sum_1^n \left(y_i - \frac{\bar{y}}{\bar{x}} x_i\right)^2 . \tag{4}$$

Some textbooks mention (but not endorse) v_2 as an alternative to v_0,

$$v_2' = \frac{1-f}{n}\left(\frac{\bar{X}}{\bar{x}}\right)^2 \frac{1}{n-1}\sum_1^n \left(y_i - \frac{\bar{y}}{\bar{x}} x_i\right)^2 . \tag{5}$$

The original motivation for $v_2' = v_2/\bar{X}^2$ as a variance estimator of the ratio

$$R = \bar{Y}/\bar{X}$$

is the unavailability of $\bar{X}$. Both v_0 and v_2 are easy to compute.

3. VARIANCE ESTIMATORS UNDER STUDY

Let $e_i = y_i - Rx_i$ be the residual from the straight line connecting $(\bar{X},\bar{Y})$ and the origin, $\tilde{e}_i = y_i - rx_i$, $r = \bar{y}/\bar{x}$, be its sample analogue. Apart from a constant, V_{appr} is the population mean of the residual square e_i^2. Estimation of V_{appr} can be viewed as the more typical problem of estimating the population mean of a new characteristic e_i^2. By taking $e_i \approx \tilde{e}_i$, v_0 can be viewed as the sample mean of e_i^2 and should be less efficient than the ratio-type estimators v_2 or

$$v_1 = \frac{\bar{X}}{\bar{x}} v_0 = \frac{1 - f}{n} \frac{\bar{X}}{\bar{x}} \frac{1}{n - 1} \sum_1^n \tilde{e}_i^2 \tag{6}$$

when x_i and e_i^2 are positively correlated. A general class of estimators

$$v_g = \left(\frac{\bar{X}}{\bar{x}}\right)^g v_0 \tag{7}$$

was proposed in Wu (1982). He proved that the leading term of $MSE(v_g)$ is minimized by

$$g_{opt} = \frac{S_{xz}\bar{X}}{S_x^2\bar{Z}}$$

$$= \text{population regression coefficient of} \tag{8}$$

$$\frac{z_i}{\bar{Z}} \text{ over } \frac{x_i}{\bar{X}},$$

where

$$z_i = e_i^2 - 2\, e_i \sum_1^N x_i e_i / \sum_1^N x_i\,,$$

S_x^2 and S_{xz} are the population x-variance and (x,z)-covariance respectively. The second term of z_i accounts for the possible nonzero intercept in the population when fitted by a straight line according to (2). A (large-sample) model-free comparison of v_2 and v_0 readily obtains. When and only when $g_{opt} > 1$, v_2 is better than v_0. It may be easier to remember and to interpret the following approximation to g_{opt} (by ignoring the second term of z_i)

$$g' = \text{population regression coefficient of}$$

$$\frac{e_i^2}{N^{-1}\Sigma_1^N e_i^2} \text{ over } \frac{x_i}{\bar{X}}. \tag{9}$$

By taking a sample analogue to g_{opt}

$$\hat{g}_{opt} = \text{sample regression coefficient of}$$

$$\frac{\tilde{z}_i}{\bar{z}} \quad \text{over} \quad \frac{x_i}{\bar{x}} , \tag{10}$$

$$\tilde{z}_i = \hat{e}_i^2 - 2 \hat{e}_i \sum_1^n x_i \hat{e}_i / \sum_1^n x_i , \quad \bar{z} = n^{-1} \sum_1^n \tilde{z}_i ,$$

we obtain an asymptotically optimal estimator $v_{\hat{g}_{opt}}$ within the class (7). Similarly we can take a sample analogue to g'

$$\tilde{g} = \text{sample regression coefficient of}$$

$$\frac{\hat{e}_i^2}{n^{-1}\Sigma_1^n \hat{e}_i^2} \quad \text{over} \quad \frac{x_i}{\bar{x}} \tag{11}$$

and obtain another estimator $v_{\tilde{g}}$.

Instead of making a ratio adjustment to the sample mean of $\hat{e}_i^2$ as in v_1, Fuller (1981) suggested a regression adjustment to v_0. Denote his estimator by

$$v_{reg} = v_0 + \frac{1-f}{n} \hat{b}_{e^2x} (\bar{X} - \bar{x}) \tag{12}$$

where

$$\hat{b}_{e^2x} = \text{sample regression coefficient of}$$

$$\hat{e}_i^2 \quad \text{over} \quad x_i .$$

By standard Taylor expansion, the leading term of $v_{\tilde{g}}$ is v_{reg} and their asymptotic behaviors should be close.

Another estimator of interest is the jackknife variance estimator

$$v_J = (1-f)\, \bar{x}^{-2} \frac{n-1}{n} \sum_1^n D_{(j)}^2 , \tag{13}$$

where $D_{(j)}$ is the difference between the ratio $(n\bar{y} - y_j)/(n\bar{x} - x_j)$ and the average of these n ratios. Royall and Cumberland (1981) and Krewski and Chakrabarty

(1981) studied the model-based and sampling properties of v_J. Note that the usual justification of jackknife is independent of a superpopulation model.

Royall and Eberhardt (1975) suggested

$$v_H = v_0 \frac{\bar{x}_c \bar{X}}{\bar{x}^2} \left(1 - \frac{C_x^2}{n}\right)^{-1} \tag{14}$$

when $\bar{x}_c$ = x-mean of non-sampled units, C_x = x-sample coefficient of variation. Later Royall and Cumberland (1978) suggested a closely related estimator

$$v_D = \frac{1 - f}{n} \frac{\bar{x}_c \bar{X}}{\bar{x}^2} \frac{1}{n} \sum_1^n \frac{\hat{e}_i^2}{1 - \frac{x_i}{n\bar{x}}} . \tag{15}$$

Both v_H and v_D are shown to be unbiased under model (2), approximately unbiased for more general variance patterns, and asymptotically equivalent to v_J.

Another variance estimator, which follows from standard least squares theory, is

$$v_L = \frac{1 - f}{n} \frac{\bar{x}_c \bar{X}}{\bar{x}^2} \frac{1}{n - 1} \sum_1^n \frac{\hat{e}_i^2}{x_i} .$$

It is unbiased under model (2) but can be seriously biased if $\mathrm{var}(y_i) = \sigma^2 x_i$ in (2) is violated (Royall and Eberhardt, 1975). Their empirical behavior has been shown to be equally bad in Royall and Cumberland (1981). For these reasons v_L will not be considered in our study.

4. <u>POPULATIONS UNDER STUDY</u>

The preceding variance estimators are compared empirically on nine populations listed in Table 1. The first six are natural populations. The original data were generously provided to us by Professors W. G. Cumberland and R. M. Royall, to whom we wish to express our sincere thanks. For more detailed description of these populations, see their 1981 paper. The last three are transformations of population 1. Their description follows. The first six populations are plotted in Royall and Cumberland (1981, p. 69-70). Though being natural populations, they are all

TABLE 1. STUDY POPULATIONS

Population	Description	x	y
1	Counties in NC, SC, and GA with 1960 white female population <100,000	Adult white female population, 1960	Breast cancer mortality, 1950-69 (white females)
2	U.S. cities with 1960 population between 100,000 and 1,000,000	Population, 1960	Population, 1970
3	Counties in NC, SC, and GA with fewer than 100,000 households in 1960	Number of households, 1960	Population, excluding residents of group quarters, 1960
4	Counties in NC, SC, and GA with fewer than 100,000 households in 1960	Number of households, 1960	Population, excluding residents of group quarters, 1970
5	National sample of short-stay hospitals with fewer than 1,000 beds	Number of beds	Number of patients discharged
6	Corporations with 1974 gross sales between one-half billion and fifty billion dollars	Gross sales, 1974	Gross sales, 1975
7	Transformation of population 1 (see (17))		
8	Transformation of population 1 (see (18))		
9	Transformation of population 1 (see (19))		

For sources of populations 1 to 6, see Royall and Cumberland (1981, p. 68).

well described by straight lines through the origin using weighted least squares. The squared residuals from the fitted line increase roughly in proportion to x. More refined models like $\alpha + \beta x$ (linear regression with intercept) and $\alpha + \beta x + rx^2$ (quadratic regression) do not differ significantly from the simpler linear-through-the origin model βx except possibly for population 5. To represent broader range of real populations, we construct populations 7, 8 and 9 from population 1 to reflect the violation of three key assumptions underlying the linear-through-the origin model (2): (i) zero intercept, (ii) $\mathrm{var}(y_i) \propto x_i$, (iii) linearity of Ey_i in x_i. More precisely, decompose the y_i value in population 1, denoted old y_i, into

$$\begin{aligned} \text{old } y_i &= Rx_i + (y_i - Rx_i) \\ &= \hat{y}_i + e_i \,. \end{aligned} \tag{16}$$

Define the new y_i value in population 7 as

$$\text{new } y_i = \text{old } y_i + \bar{Y} \;; \tag{17}$$

for population 8,

$$\text{new } y_i = \hat{y}_i + kx_i e_i \,, \tag{18}$$

with $k = S_x^{-1}$ and all units except two have $y_i > 0$; for population 9,

$$\text{new } y_i = c_0\left[c_1 - e^{\beta(x_i - \bar{X})}\right] + e_i \,, \tag{19}$$

where $\beta = S_x^{-1}$, $c_0 = S_y$, $c_1 = 0.1 + \exp\left[\beta\left(\max\limits_{1 \le i \le N} x_i - \bar{X}\right)\right]$ so that $y_i > 0$ for all i.

Populations 1, 7, 8 and 9 are shown below.

Some characterisics of the populations are given in Table 2. Note that x and y are highly correlated (> 0.94) for populations 1-4, 6, 7. The x and y of the transformed populations 8 and 9 are less correlated. Another point to observe is that v_{appr} can be smaller or larger than MSE for sample size 32. There is no systematic pattern in the percent underestimate or overestimate (last

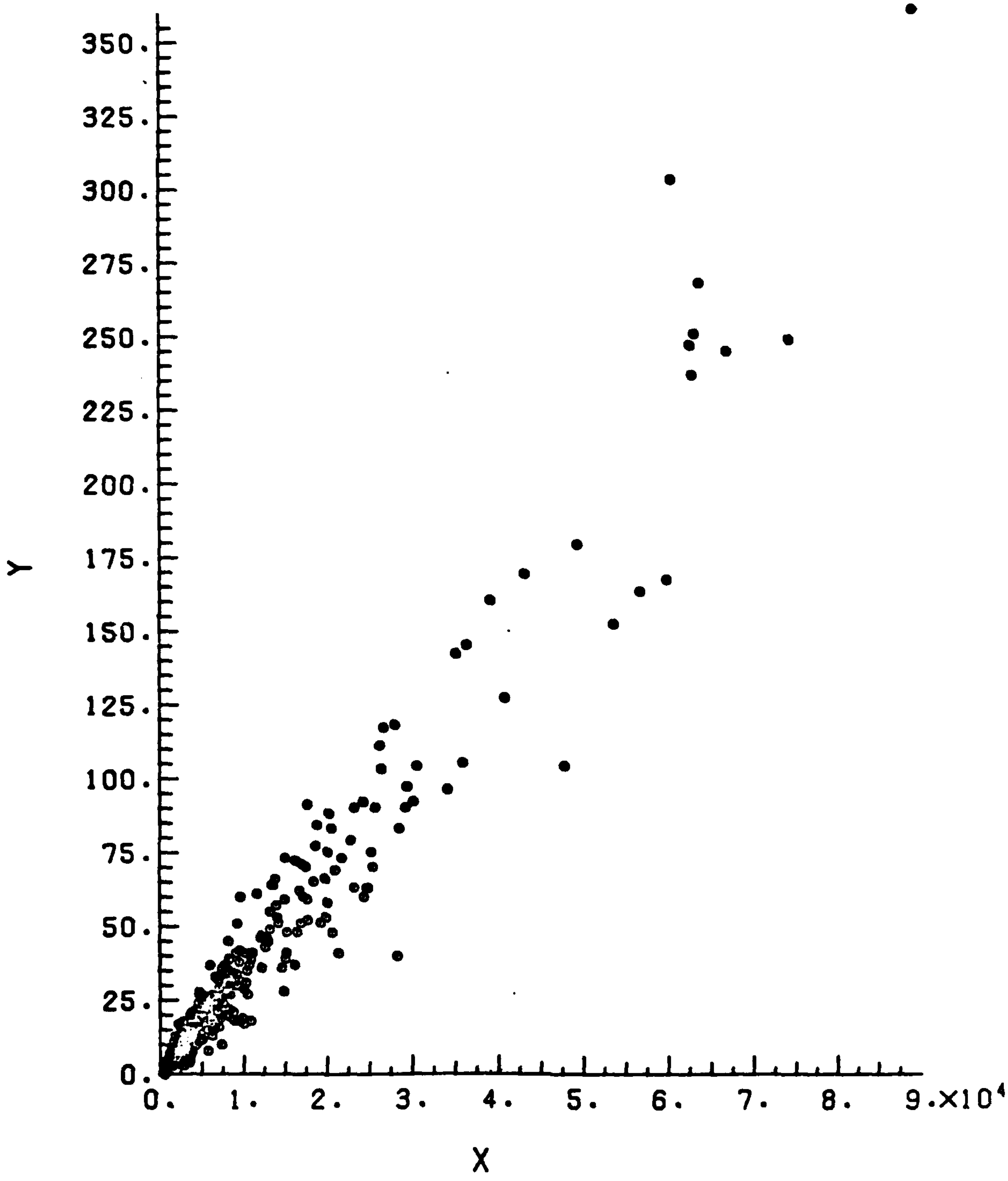


Figure 1. Population (1)

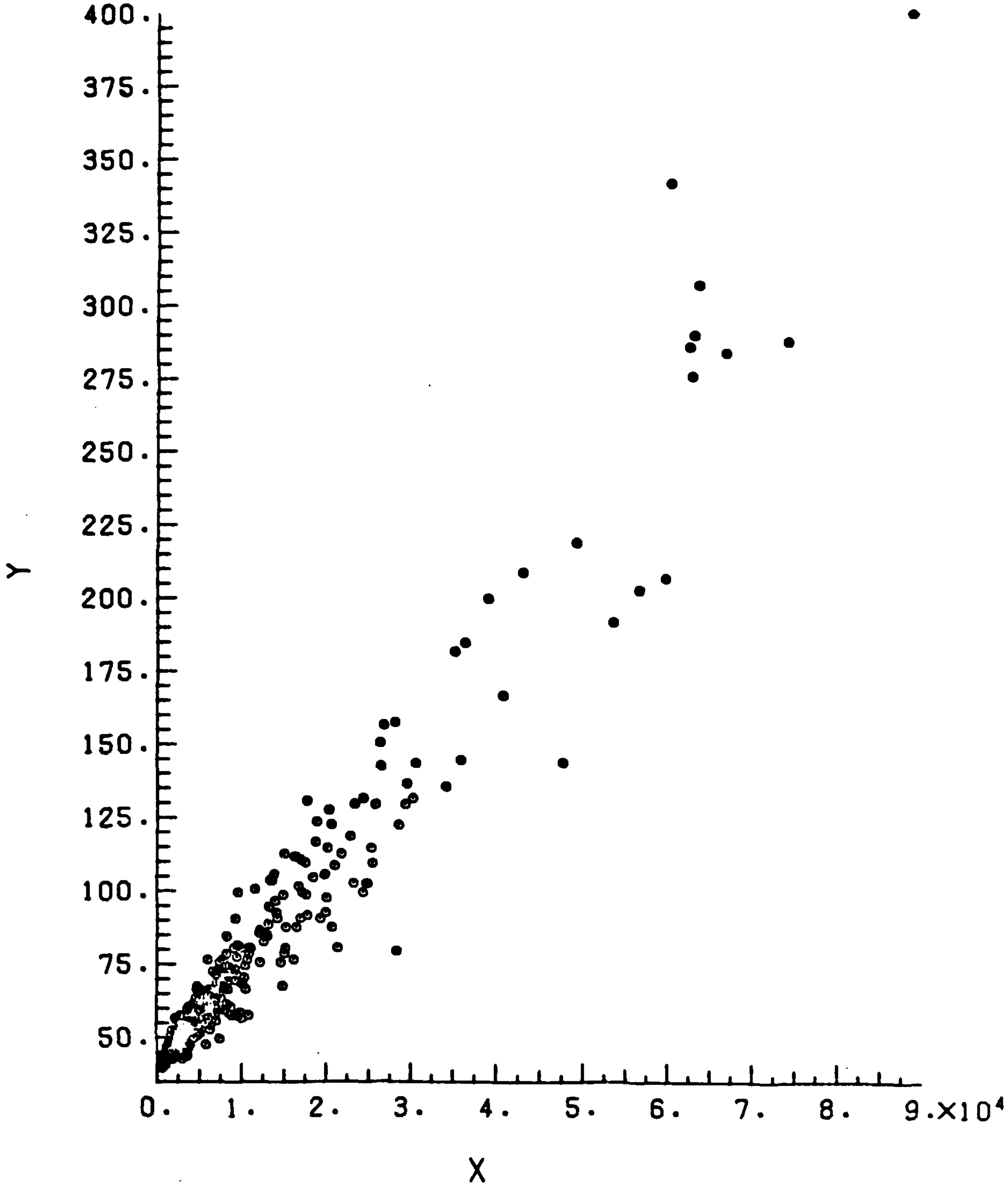


Figure 2. Population (7)

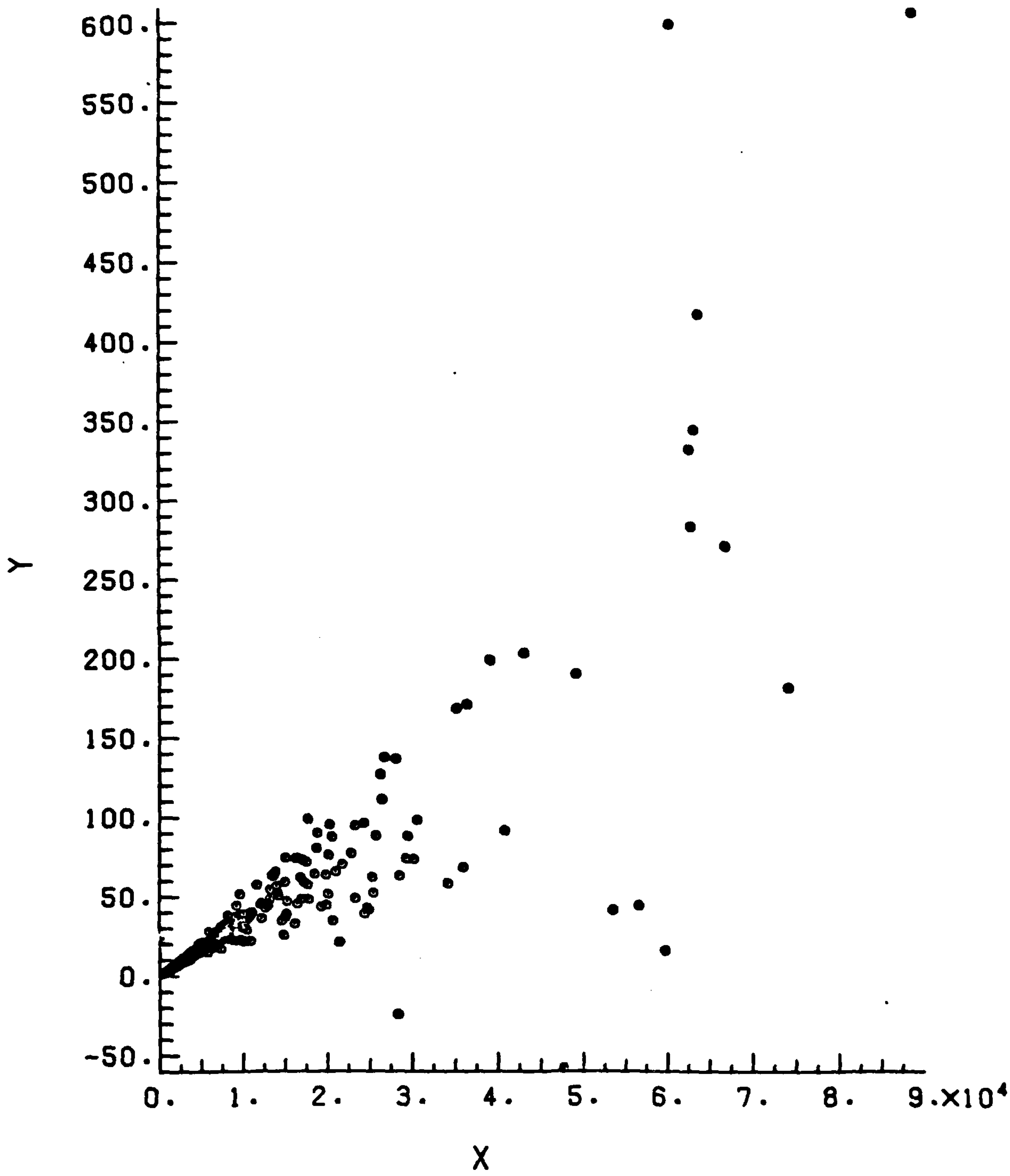


Figure 3. Population (8)

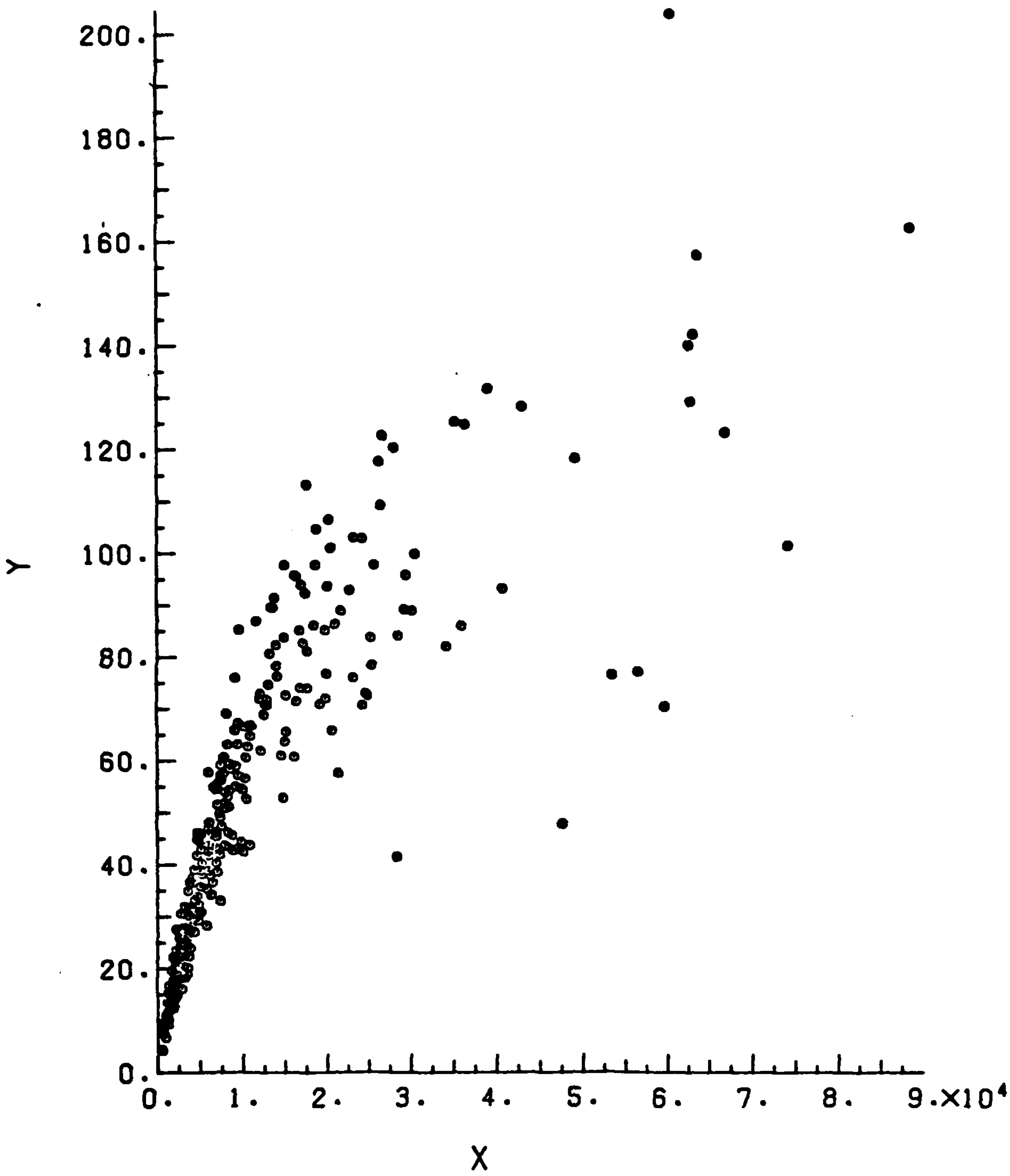


Figure 4. Population (9)

TABLE 2. SOME POPULATION CHARACTERISTICS

Population	N	$\rho(X,Y)^a$	$MSE(\hat{\bar{y}}_R)^b$	$v_{appr}(\hat{\bar{y}}_R)$	$\frac{100(v_{appr} - MSE)}{MSE}$
1	301	0.967	4.61	4.71	2.2
2	125	0.947	113.9×10^6	111.9×10^6	-1.8
3	304	0.998	33.6×10^4	32.4×10^4	-3.6
4	304	0.982	230.9×10^4	264.5×10^4	14.6
5	393	0.911	1941.0	1968.0	1.4
6	331	0.997	30.7×10^{14}	35.7×10^{14}	16.3
7	301	0.967	77.7	69.0	-11.2
8	301	0.805	44.3	47.6	7.4
9	301	0.824	44.6	43.0	-3.6

a. Correlation between x and y populations.
b. Based on 1000 simulated samples.

column of Table 2) in v_{appr} of MSE with only three of them over ten percent. This is quite different from Rao (1968), where he found that, for smaller sample sizes $n = 4, 6, 8, 12$, v_{appr} consistently underestimates MSE with average percent underestimates ranging between 12% and 17%. The discrepancy is explained in part by the difference in sample sizes. More importantly, Rao's computation of v_{appr} and MSE is apparently based on a particular superpopulation model while ours is model free.

5. RESULTS

We draw 1000 simple random samples of size $n = 32$ from each population. For each sample we calculate the ratio estimate $\hat{\bar{y}}_R$ and the variance estimates v_0, v_1, v_2, $v_{\hat{g}_{opt}}$, $v_{\tilde{g}}$, v_{reg}, v_J, v_H, v_D and $v_{g_{opt}}$. Note $v_{g_{opt}}$ is not really an estimator since g_{opt} depends on the whole population. We include it here to see how the asymptotic results in Wu (1982) (or §3) predict the actual performance for sample size 32. The $MSE(\bar{y}_R)$ in Table 2 is calculated as $1000^{-1} \sum_1^{1000} (\hat{\bar{y}}_R - \bar{Y})^2$ over the 1000 simulated samples. For each variance estimator v, its bias $bias(v)$ is calculated as $1000^{-1} \sum_1^{1000} v - MSE(\hat{\bar{y}}_R)$ over the same 1000 samples, and its root mean-square error $\sqrt{MSE}(v)$ as $(1000^{-1} \sum_1^{1000} (v - MSE(\hat{\bar{y}}_R))^2)^{1/2}$ over the same 1000 sample. Results are given in Table 3.

We first summarize the root mean square error behavior of the ten estimators in Table 3 as follows.

(i) The asymptotically optimal estimator $v_{g_{opt}}$ (pretending g_{opt} is available) is the best or nearly the best estimator in terms of minimizing MSE, as well predicted by the asymptotic result of Wu (1982).

(ii) Among v_0, v_1, v_2, the best performer is consistently the one closer to g_{opt}. For example, $g_{opt} = 1.59$ in population 1 is closer to 2 and thus v_2 has smaller $\sqrt{MSE}$ (2.20) than those (2.26 and 2.73) of v_1 and v_0 respectively. This is again predicted by the asymptotic result of Wu (1982, §2.2).

Table 3. Root mean-square error and bias* of variance estimators

variance estimator	Population 1	2	3	4	5	6	7	8	9
v_0	2.73 (-0.29)	54.3 (-0.7)	15.7 (-5.6)	262 (-13.5)	819 (-45)	24.9 (-0.6)	22.7 (-12.4)	42.1 (- 7.3)	18.8 (-6.9)
v_1	2.26 (-0.43)	53.0 (-1.2)	13.9 (-6.4)	224 (-27.8)	731 (-71)	18.6 (-1.9)	24.1 (-10.6)	36.8 (-10.2)	17.3 (-7.6)
v_2	2.20 (-0.39)	56.6 (-0.2)	14.1 (-6.0)	214 (-32.1)	735 (-66)	17.1 (-1.0)	32.4 (- 6.2)	35.0 (-11.9)	18.1 (-7.0)
$v_{\hat{g}_{opt}}$	2.26 (-0.44)	53.5 (-1.4)	14.7 (-4.3)	228 (-26.7)	755 (-65)	18.7 (-1.1)	23.6 (-10.4)	35.8 (-11.0)	18.0 (-5.5)
$v_{\tilde{g}}$	2.33 (-0.40)	53.5 (-1.2)	15.7 (-4.8)	243 (-25.4)	766 (-70)	18.6 (-1.4)	36.2 (- 4.3)	37.7 (-10.5)	22.3 (-4.3)
v_{reg}	2.20 (-0.45)	52.3 (-1.3)	14.0 (-6.2)	209 (-35)	721 (-75)	17.4 (-2.3)	43.5 (- 8.8)	35.2 (-11.5)	35.3 (-7.2)
v_H	2.27 (-0.25)	60.5 (3.4)	14.4 (-5.0)	220 (-25.7)	760 (-7)	17.8 (0.1)	34.6 (- 3.3)	35.6 (-11.0)	18.5 (-5.7)
v_D	2.46 (-0.09)	59.5 (2.6)	15.8 (-2.9)	251 (- 9.8)	782 (1)	21.3 (2.4)	38.9 (1.5)	39.0 (- 7.6)	21.2 (-1.9)
v_J	2.76 (0.23)	58.3 (5.2)	18.6 (0.9)	303 (18.6)	829 (72)	28.0 (6.9)	45.4 (9.3)	45.0 (- 2.2)	26.2 (4)
$v_{g_{opt}}$	2.17 (-0.43)	53.3 (-1.1)	13.7 (-6.4)	218 (-30.8)	722 (-72)	17.1 (-1.5)	22.6 (-12.4)	35.6 (-12.4)	17.2 (-7.6)
g_{opt}	1.59	1.18	1.24	2.46	1.60	1.72	0.06	2.80	1.05
unit	10^0	10^6	10^4	10^4	10^0	10^{14}	10^0	10^0	10^0

*Bias given inside the parenthesis

(iii) The estimators $v_{\hat{g}}$, $v_{\tilde{g}}$ and v_{reg} are asymptotically equivalent and are close to $v_{g_{opt}}$. They all give small $\sqrt{MSE}$. It is somewhat surprising that v_{reg} does as well as $v_{g_{opt}}$ and better than $v_{\hat{g}}$ and $v_{\tilde{g}}$ on populations 1-6 and 8. Reasons for the poor performance of v_{reg} on populations 7 and 9 are not known. The estimators $v_{\hat{g}}$, $v_{\tilde{g}}$ and v_H are more stable in that they perform reasonably well for all the populations.

(iv) The jackknife variance estimator v_J is the worst in terms of MSE. The instability of v_J was also reported in Rao and Rao (1971), Rao and Kuzik (1974), Krewski and Chakrabarty (1981). The performance of v_D is not good either.

The bias of each variance estimator is given inside the parenthesis in Table 3. The results are summarized as follows.

(i) The bias is usually a small proportion (say, $<$ 30%) of the total $\sqrt{MSE}$ with a few exceptions for populations 3, 7, 8, 9.

(ii) The estimators v_0, v_1, v_2 are consistently downward biased for estimating the MSE. The estimators $v_{\hat{g}}$, $v_{\tilde{g}}$ and v_{reg}, being close to one of v_0, v_1 or v_2, are consistently downward biased. Another intriguing phenomenon: among v_0, v_1, v_2, those with smaller $\sqrt{MSE}$ tend to have bigger (in magnitude) bias.

(iii) The estimator v_J is almost always upward biased, while v_H and v_D exhibit no systematic pattern.

The downward biasedness of v_0 was noted in Rao (1968), Rao and Rao (1971). And the upward biasedness of v_J was noted in Rao and Rao (1971). Both are exact analytic results whose validity depends on some particular superpopulation models. Model-free (but asymptotic) results on the bias of v_0, v_1, v_2, v_J, v_H have been obtained by the first author. They will appear soon.

In estimating the population mean the purpose of variance estimation is rather for assessing the variability of the ratio estimator than for estimating the variance itself. A more interesting and relevant criterion is the behavior of the associated confidence interval. For each variance estimator v and each simulated sample, we consider the t-statistic

$$t = \frac{\hat{\bar{y}}_R - \bar{Y}}{\sqrt{v}} , \tag{20}$$

and the $(1 - \alpha)$ confidence interval for estimating $\bar{Y}$

$$(\hat{\bar{y}}_R - t_{\alpha/2}(31)\sqrt{v},\ \hat{\bar{y}}_R + t_{\alpha/2}(31)\sqrt{v}) \tag{21}$$

where $t_{\alpha/2}(31)$ is the upper $\alpha/2$ point of the t-distribution with d.f. = 31. The Monte-Carlo coverage probability of the confidence interval (21), given in Table 4, is calculated as the percentage of the 1000 intervals (21) that cover $\bar{Y}$. The bias, standard deviation and coefficient of skewness of the associated t-statistic, given in the last three columns of Table 4, are based on the 1000 t-values (20).

We now summarize Table 4 in three parts: I. normality of t-statistic, II. width of t-interval, III. reliability of t-interval in terms of the closeness of its Monte Carlo coverage probability to the nominal one.

I. Except for populations 4 and 9, the bias is close to zero, the s.d. close to one and the coefficient of skewness close to zero. Typically the t-statistic associated with the estimator v_0 is not normal, especially with its large coefficient of skewness.

II. Since the squared length of the t-interval is proportional to the expected value of v, from $E(v)$ = bias of v + MSE, we can use the bias entry of Table 3 in assessing the width of t-interval. Since v_J has positive bias, the corresponding t-interval is wider. Similarly v_0, v_1, v_2, $v_{\hat{g}}$, $v_{\tilde{g}}$, v_{reg} all have negative bias. Their t-intervals are shorter. The intervals associated with v_H and v_D are in between the two extremes.

III. (i) Generally the coverage probability is lower than the nominal level $1 - \alpha$. This may in part be explained by the negative bias of v in most cases (except v_J and some cases of v_H and v_D) The

Table 4. Coverage probabilities of the t-intervals $(\hat{\hat{y}}_R - c\sqrt{v},\ \hat{\hat{y}}_R + c\sqrt{v})$, $c = t_{\frac{\alpha}{2}}(31)$, and descriptive statistics of $t = \dfrac{\hat{\hat{y}}_R - \bar{Y}}{\sqrt{v}}$ based on 1000 simple random samples of $n = 32$

Population	variance estimator	nominal coverage probability $(1 - \alpha) \times 100\%$ 99	95	90	80	70	bias	s.d. (of t)	coefficient of skewness
	v_0	97.1	91.5	84.8	74.5	64.6	-0.011	1.23	0.066
	v_1	97.7	92.1	86.3	75.2	64.6	-0.012	1.18	0.035
	v_2	97.8	93.0	87.1	76.0	64.7	-0.012	1.16	0.030
	$v_{\hat{g}_{opt}}$	97.3	92.6	86.5	75.0	64.7	-0.009	1.18	0.051
1	$v_{\tilde{g}}$	97.7	92.9	86.7	75.0	64.6	-0.006	1.19	0.049
	v_{reg}	97.7	92.5	86.7	75.1	64.6	-0.009	1.18	0.040
	v_H	97.8	93.2	87.8	76.7	65.5	-0.012	1.14	0.030
	v_D	97.8	93.4	88.0	77.2	66.1	-0.012	1.13	0.034
	v_J	97.9	94.1	88.6	78.6	67.7	-0.013	1.10	0.033

2	v_0	96.3	90.4	85.7	75.9	67.5	-0.17	1.22	-0.78
	v_1	96.1	91.0	86.0	76.8	68.1	-0.18	1.21	-0.83
	v_2	95.7	91.4	86.4	76.4	68.2	-0.19	1.20	-0.87
	$v_{\hat{g}_{opt}}$	96.0	90.7	85.7	76.1	67.6	-0.18	1.21	-0.81
	$v_{\tilde{g}}$	95.9	91.1	85.8	76.2	67.4	-0.18	1.21	-0.81
	v_{reg}	95.9	91.0	85.9	76.6	67.8	-0.18	1.21	-0.82
	v_H	95.7	91.8	86.6	77.2	68.7	-0.19	1.19	-0.88
	v_D	95.7	91.6	86.5	77.1	68.6	-0.19	1.19	-0.86
	v_J	96.0	92.3	87.0	77.9	68.8	-0.18	1.17	-0.88
3	v_0	93.1	86.7	80.7	71.2	63.4	0.23	1.42	1.02
	v_1	94.6	88.8	80.9	71.6	62.9	0.13	1.31	0.57
	v_2	96.9	88.2	82.5	71.1	62.3	0.04	1.25	0.13
	$v_{\hat{g}_{opt}}$	96.2	90.0	83.5	73.1	64.6	0.12	1.24	0.55
	$v_{\tilde{g}}$	96.5	89.1	82.4	72.2	63.1	0.08	1.25	0.41
	v_{reg}	95.2	88.7	81.1	71.1	62.6	0.11	1.30	0.51
	v_H	97.4	88.8	83.1	72.0	62.7	0.03	1.23	0.08
	v_D	97.5	89.9	83.7	73.3	64.2	0.04	1.21	0.11
	v_J	97.8	91.2	85.6	75.2	66.3	0.05	1.16	0.21

p. 69-70). Though being natural populations, they are all

4	v_0	85.8	77.1	72.1	63.7	55.9	-0.79	1.69	-0.95
	v_1	87.3	77.7	72.6	63.7	55.0	-0.74	1.59	-0.79
	v_2	88.6	78.9	72.3	63.9	54.6	-0.69	1.53	-0.71
	$v_{\hat{g}_{opt}}$	87.5	77.8	72.4	63.2	55.2	-0.74	1.60	-0.82
	$v_{\tilde{g}}$	87.3	77.9	72.1	63.3	55.0	-0.73	1.59	-0.78
	v_{reg}	87.0	77.9	72.2	63.3	54.7	-0.73	1.59	-0.77
	v_H	89.0	79.9	73.1	64.5	55.2	-0.68	1.51	-0.71
	v_D	89.7	79.9	73.9	65.4	55.7	-0.68	1.48	-0.74
	v_J	90.6	81.5	75.4	66.5	57.9	-0.66	1.43	-0.77
5	v_0	97.3	92.9	88.0	77.5	67.3	0.12	1.14	0.40
	v_1	97.8	93.4	89.2	77.9	67.5	0.09	1.11	0.27
	v_2	97.9	94.0	90.1	77.5	67.7	0.06	1.10	0.15
	$v_{\hat{g}_{opt}}$	97.9	93.7	89.4	77.3	67.5	0.08	1.11	0.23
	$v_{\tilde{g}}$	97.8	93.5	89.2	77.7	67.6	0.07	1.11	0.21
	v_{reg}	97.8	93.4	89.4	77.9	67.6	0.08	1.11	0.25
	v_H	98.2	94.3	90.3	78.6	68.3	0.06	1.08	0.14
	v_D	98.1	94.3	90.1	79.1	68.3	0.06	1.08	0.14
	v_J	98.2	94.6	90.9	78.4	68.9	0.06	1.07	0.16

6	v_0	97.3	92.2	84.3	73.4	65.9	-0.001	1.19	0.055
	v_1	98.6	93.3	86.4	75.5	67.1	0.001	1.13	0.023
	v_2	98.4	93.4	87.6	78.0	67.8	0.021	1.10	0.066
	$v_{\hat{g}_{opt}}$	97.8	93.3	86.2	75.6	68.3	0.011	1.14	0.062
	$v_{\tilde{g}}$	97.9	93.8	86.1	76.1	67.7	0.021	1.13	0.075
	v_{reg}	98.3	93.8	86.2	75.4	67.5	0.013	1.13	0.035
	v_H	98.5	93.8	87.9	79.1	68.9	0.023	1.08	0.078
	v_D	98.8	94.6	88.4	80.0	70.6	0.020	1.06	0.080
	v_J	99.1	95.8	89.8	81.7	72.0	0.015	1.01	0.072
7	v_0	95.7	91.8	87.8	79.6	70.5	0.24	1.23	1.46
	v_1	97.5	92.7	88.2	80.0	70.5	0.13	1.12	0.63
	v_2	98.4	93.7	88.4	80.1	70.1	0.03	1.08	-0.11
	$v_{\hat{g}_{opt}}$	95.7	92.1	88.7	81.5	71.6	0.25	1.21	1.60
	$v_{\tilde{g}}$	99.0	94.5	89.7	81.2	71.7	0.06	1.04	0.16
	v_{reg}	98.1	93.2	88.4	80.4	70.6	0.10	1.09	0.47
	v_H	98.3	94.0	89.2	80.5	71.3	0.01	1.06	-0.19
	v_D	98.6	94.2	89.5	81.0	72.3	0.01	1.04	-0.20
	v_J	98.7	94.7	90.7	82.8	73.6	0.02	1.01	-0.11

8	v_0	95.7	87.1	78.1	64.5	53.3	-0.25	1.36	-0.01
	v_1	97.2	88.9	79.7	65.5	53.2	-0.23	1.31	0.02
	v_2	97.5	88.6	80.8	66.8	52.8	-0.21	1.28	0.06
	$v_{\hat{g}_{opt}}$	97.4	89.3	80.7	67.1	53.8	-0.21	1.28	0.01
	$v_{\tilde{g}}$	97.6	89.1	80.5	67.9	54.5	-0.20	1.27	0.06
	v_{reg}	96.8	87.8	79.4	66.5	52.5	-0.22	1.31	-0.05
	v_H	97.9	88.7	81.6	67.8	53.9	-0.20	1.26	0.06
	v_D	98.3	90.4	84.0	69.8	55.6	-0.20	1.21	0.05
	v_J	98.7	93.6	81.6	67.8	53.9	-0.20	1.15	0.03
9	v_0	91.2	86.3	82.5	75.3	67.3	0.50	1.67	2.40
	v_1	93.1	86.9	82.6	74.9	66.5	0.36	1.46	1.57
	v_2	94.6	88.1	82.2	74.2	66.6	0.25	1.32	0.88
	$v_{\hat{g}_{opt}}$	93.1	87.8	83.7	76.5	68.4	0.38	1.40	1.64
	$v_{\tilde{g}}$	96.2	89.5	83.7	74.6	67.0	0.22	1.26	0.87
	v_{reg}	93.8	88.0	82.5	74.4	66.7	0.30	1.39	1.34
	v_H	94.9	88.7	82.6	74.6	67.6	0.23	1.30	0.81
	v_D	95.4	89.2	83.7	75.4	68.9	0.23	1.27	0.85
	v_J	95.7	90.5	85.5	76.8	70.3	0.23	1.23	0.96

discrepancy is most serious in population 4 where the t-statistics are "abnormal", and is serious in population 8 for $1 - \alpha = 0.7, 0.8, 0.9$.

(ii) The estimator v_0 is least reliable in that the actual coverage probability is far off the nominal level. The most reliable one among the nine estimators is the jackknife v_J with v_D as the second best. On populations 1 and 5-9 v_D performs as well or nearly as well as v_J. The performance of v_H is comparable to v_D (and sometimes v_J) on populations 1, 2, 4, 5 and 7. Other estimators v_1, v_2, $v_{\hat{g}}$, $v_{\bar{g}}$, v_{reg} are comparable among each other but trail slightly behind v_H.

(iii) Among v_0, v_1, v_2, v_2 is the best, v_1 the middle and v_0 the worst for most cases. A partial explanation is that v_2 is the only one among the three that is asymptotically equivalent to v_J, the best performer.

The excellent performance of v_J might be explained by the large expected value Ev_J, or equivalently the width of the associated t-interval. In the same spirit, could the better performance of v_2 relative to v_1 and v_1 to v_0 be attributed to any similar behavior in their t-intervals? As remarked in II they are all short intervals, but from Table 3 the biases of v_0, v_1, v_2 do not exhibit any clear-cut pattern to support such claim. We do not believe that the length of the interval alone explains the difference.

One obvious thing to observe from comparing Tables 3 and 4 is that, estimators like $v_{\hat{g}}$, $v_{\bar{g}}$, v_{reg} that perform well for estimating $MSE(\hat{\bar{y}}_R)$ do not fare very well for giving reliable interval estimate. On the other hand, v_J, though having very large mean square error for estimating $MSE(\hat{\bar{y}}_R)$, is extremely good in giving reliable interval estimate. Perhaps the only consistent conclusion from Table 3 and 4 is that v_0 fares poorly in both criteria. Since a variance estimator is primarily judged by the quality of the associated interval estimator, an important question thus arises: What properties of v as a point estimator will provide a good guide in judging its performance as an interval estimator?

We propose to take $\bar{x}$ as an <u>ancillary</u> statistic and draw inference by conditioning on $\bar{x}$. More precisely an estimate of the conditional mean square error $MSE(\hat{\bar{y}}_R|\bar{x})$ should be used in the interval estimate of $\bar{Y}$. In the context of maximum likelihood estimation Efron and Hinkley (1978) proposed a version of conditional variance (given an appropriate ancillary statistic) for constructing reliable interval estimate.

To see how different variance estimators perform in tracking $MSE(\hat{\bar{y}}_R|\bar{x})$, we divide the 1000 samples into 20 groups of 50 samples according to the order of the $\bar{x}$ values. For each group we calculate the average of $\bar{x}$, $\sum_1^{50} \bar{x}/50$, the conditional MSE of $\hat{\bar{y}}_R$ within the group, $\sum_1^{50} (\hat{\bar{y}}_R - \bar{Y})^2/50$ and the averages of each of the nine estimates, $\bar{v}_0 = \sum_1^{50} v_0/50$, etc. We then plot the values of $N\sqrt{MSE}$, $N\sqrt{\bar{v}_0}$, $N\sqrt{\bar{v}_1}$, and so on, against the average values of $\bar{x}$ (the factor N is to make our plots comparable to those of Royall and Cumberland (1981).) To save space we only show the plots for populations 2 and 3 in Figures 5 and 6. Plots for other populations exhibit similar patterns. In Figures 5 and 6, the trajectories of $\sqrt{\bar{v}_{\tilde{g}}}$, $\sqrt{\bar{v}_{reg}}$, $\sqrt{\bar{v}_D}$ are omitted because they are too close to the trajectories of $\sqrt{\bar{v}_{\hat{g}}}$ and $\sqrt{\bar{v}_H}$ respectively. One can see that $\sqrt{\bar{v}_H}$, $\sqrt{\bar{v}_J}$ and $\sqrt{\bar{v}_2}$ seem to track the conditional $\sqrt{MSE}$ (trajectory) better than $\sqrt{\bar{v}_0}$, $\sqrt{\bar{v}_1}$ and $\sqrt{\bar{v}_{\hat{g}}}$. Such a visual comparison of trajectories is somewhat arbitrary and imprecise. Instead we consider a measure of distance (22) between the $\sqrt{MSE}$-trajectory and any $\sqrt{\bar{v}}$-trajectory given by

$$\left\{\frac{1}{20} \sum_1^{20} \left(\sqrt{\bar{v}} - \sqrt{MSE}\right)^2\right\}^{1/2}, \qquad (22)$$

where the summation is over the twenty groups of values. If the distance measure for a variance estimator v is smaller, we say that v is closer to the conditional MSE of

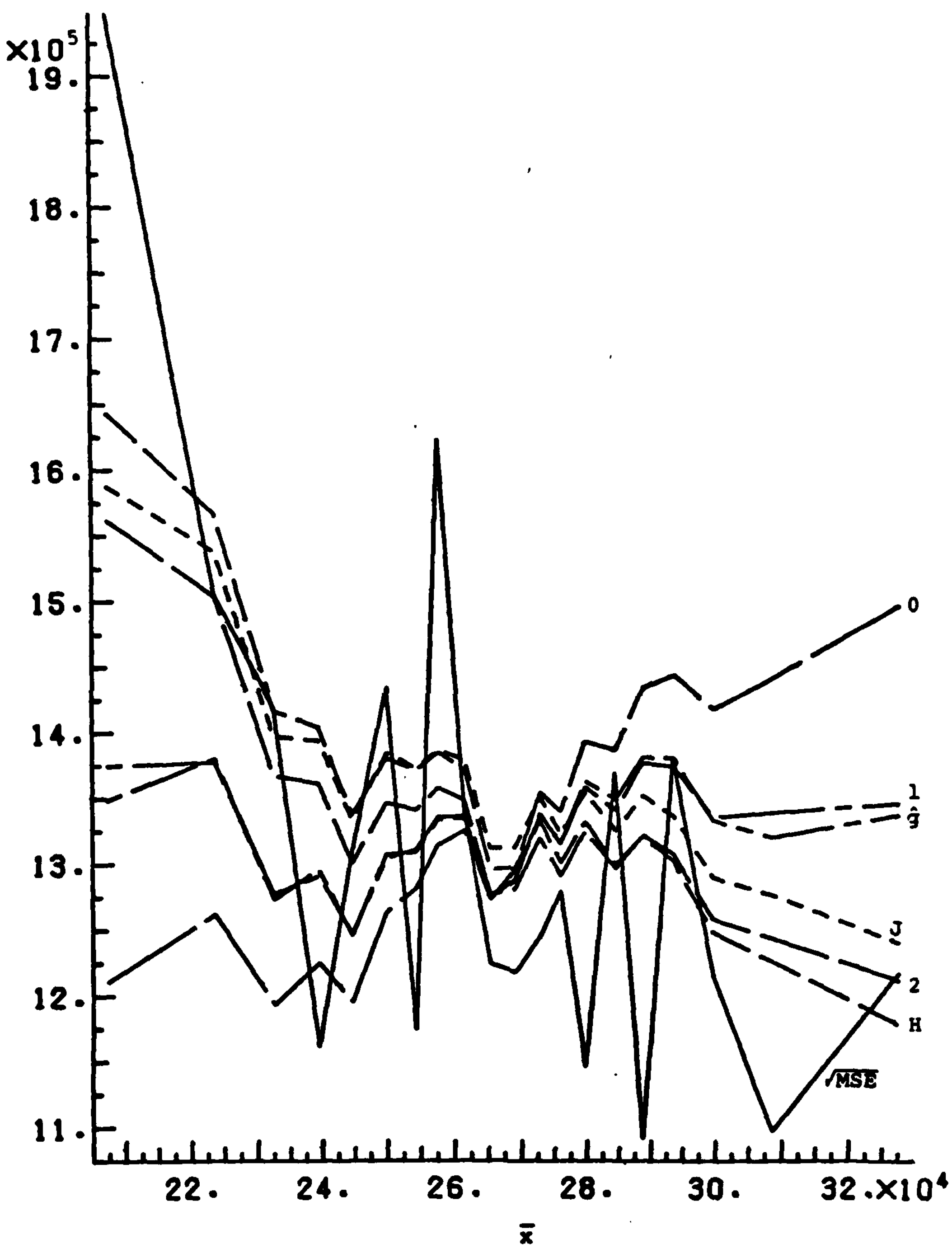


Figure 5. Curves $\sqrt{MSE}$, $\sqrt{v_0}$, $\sqrt{v_1}$, $\sqrt{v_2}$, $\sqrt{v_{\hat{g}}}$, $\sqrt{v_J}$, $\sqrt{v_H}$ for population (2).

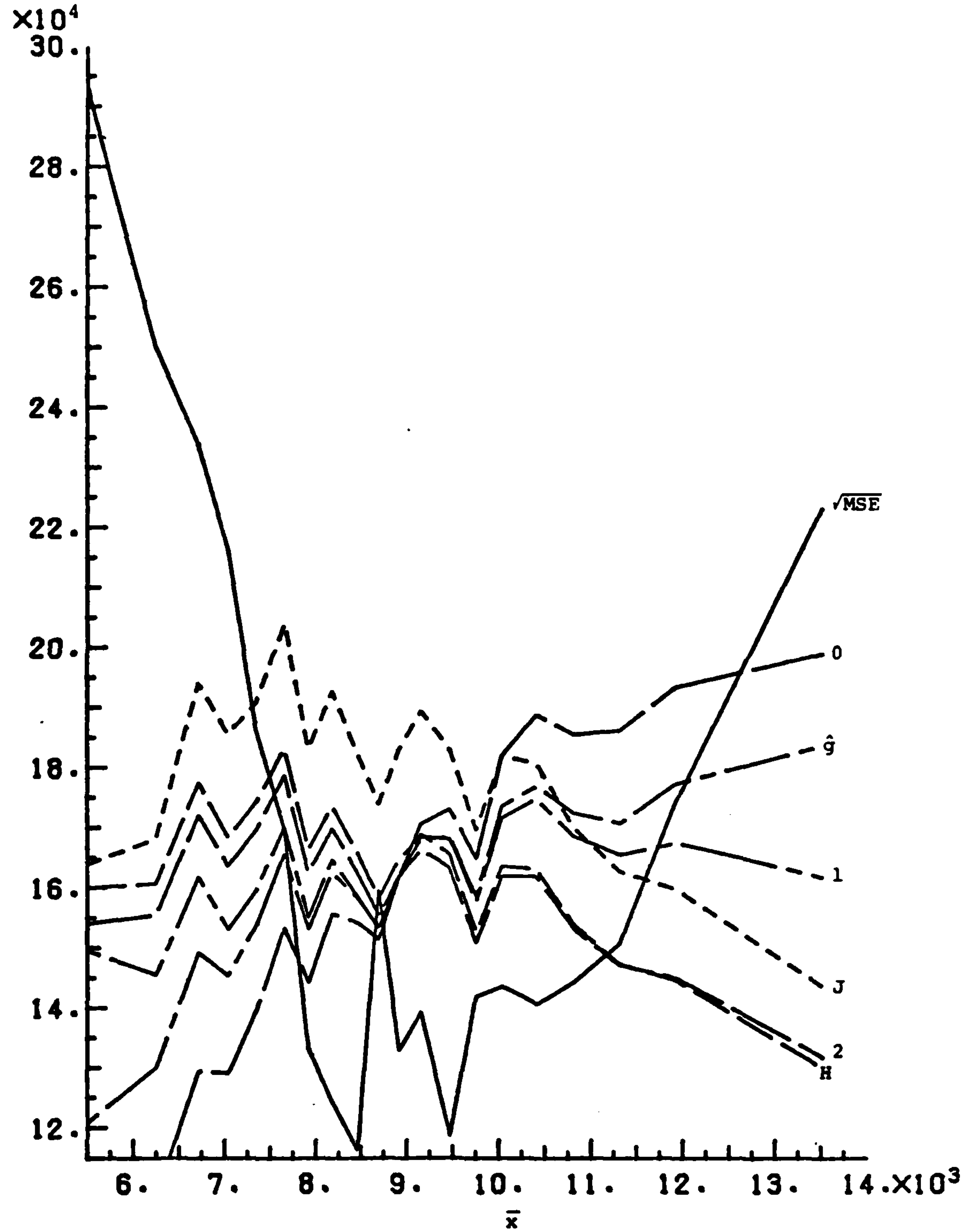


Figure 6. Curves $\sqrt{MSE}$, $\sqrt{v_0}$, $\sqrt{v_1}$, $\sqrt{v_2}$, $\sqrt{v_{\hat{g}}}$, $\sqrt{v_J}$, $\sqrt{v_H}$ for population (3).

the ratio estimator. In Table 5 we list such values for nine populations and nine estimators.

From Table 5 we can roughly rank the performance of the nine estimators as

$$v_H, v_D > v_J, v_2, v_{\tilde{g}} > v_{\hat{g}}, v_{reg} > v_1 > v_0 ,$$

where ">" means "better than". Again v_0 is the worst, v_1 the second worst. v_H and v_D are slightly better than v_J, v_2 and $v_{\tilde{g}}$. And $v_{\hat{g}}$, v_{reg} are the mediocre performers. This is in general agreement with the results of Table 4. We are thus led to the tentative conclusion that

> "Variance estimators that estimate the conditional MSE of the ratio estimator better tend to give more reliable interval estimates of the population mean."

It should be possible to justify theoretically this statement at least by assuming a reasonable superpopulation model between y and x.

The apparent contradiction between the unconditional behavior of the variance estimators as point estimators of the MSE of the ratio estimator (Table 3) and the reliability of the associated interval estimators of the population mean (Table 4) is now happily resolved. If the inference is made conditional on an appropriate ancillary statistic, in this case $\bar{x}$, good performance of an estimator for estimating the conditional variance often points to good performance of the corresponding interval estimator. Therefore our empirical finding lends further support to the work of Efron and Hinkley (1978), although ours is for finite populations and theirs for infinite populations and parametric models.

6. CONCLUSIONS AND FURTHER REMARKS

Based on the empirical study in §4 and §5 and the theoretical discussion in §3, we arrive at the following conclusions.

1) The estimator v_0, (4), is the poorest among the nine estimators considered in the paper. Its t-intervals are not reliable and it does not estimate either the MSE or the conditional MSE of $\hat{\bar{y}}_R$ well. However it is the most

Table 5. Distance, formula (22), between $\sqrt{MSE}$-trajectory and $\sqrt{\bar{v}}$-trajectory

variance estimator	Population 1	2	3	4	5	6	7	8	9
v_0	0.528	1998	227	339	8.90	15888	5.62	1.60	3.79
v_1	0.354	1535	197	225	6.85	9916	5.31	1.31	3.55
v_2	0.254	1170	171	184	5.30	7477	5.02	1.21	3.32
v_{reg}	0.325	1532	186	200	6.45	8933	5.19	1.21	3.40
$v_{\hat{g}_{opt}}$	0.320	1559	172	181	6.05	9837	5.54	1.16	3.42
$v_{\tilde{g}}$	0.297	1495	160	176	6.01	9820	4.91	1.13	3.05
v_H	0.241	1118	168	176	5.20	7579	5.04	1.14	3.32
v_D	0.232	1118	170	155	5.38	7902	5.36	0.96	3.39
v_J	0.240	1221	179	166	5.20	10181	5.15	0.81	3.55
unit	10^0	10^6	10^4	10^4	10^0	10^{14}	10^0	10^0	10^0

commonly recommended estimator in virtually every textbook on sampling. (In fact some well-known textbooks do not even mention the better estimator v_2.) The "de-mystification" of v_0 is probably the most useful of all the recommendations made in our paper.

2) Among v_0, v_1, v_2, v_2 is better than v_1 and v_1 better than v_0 for giving reliable t-intervals. The performance of v_0, v_1 and v_2 for estimating MSE depends on the underlying populations and has no direct bearing on the performance of interval estimates.

3) If more complicated computations are allowed (such may be an issue for large scale surveys), we have more choices. The jackknife v_J gives very reliable t-intervals and v_H, v_D are almost as good. Note that for large samples, all three estimators are close to v_2, but not to any other v_g, $g \neq 2$. The reason that v_J does so poorly for estimating MSE is because it estimates the conditional MSE well, and typically the conditional MSE varies greatly with $\bar{x}$. This instability of v_J for estimating the unconditional MSE has also been reported in previous papers but should not concern us any more.

4) The estimators $v_{\hat{g}}$, $v_{\bar{g}}$ and v_{reg} are asymptotically equivalent. They are good for estimating the unconditional MSE but are mediocre for giving reliable t-intervals.

We emphasize that reliable t-intervals seems to be related to the good performance of v for estimating the conditional MSE. The problem of choosing a proper ancillary statistic and making inference condit ·nal on it is an important one in the theory of survey sampling.

Encouraged by the relatively good performance of v_2 over v_0, we have considered the variance estimation problem in other settings. For the regression estimator under simple random sampling,

$$\bar{y}_{\ell r} = \bar{y} + b(\bar{X} - \bar{x}), \quad b = \frac{s_{xy}}{s_x^2}$$

the typical estimator of $\mathrm{Var}(\bar{y}_{\ell r})$ is (Cochran, 1977, p. 195)

$$\widehat{\mathrm{var}} = \frac{1 - f}{n} \frac{1}{n - 2} \sum_{1}^{n} (y_i - \bar{y} - b(x_i - \bar{x}))^2 ,$$

which is the sample analogue of the approximate variance of $\bar{y}_{\ell r}$. It is natural to consider the following class of estimators

$$\left(\frac{\bar{X}}{\bar{x}}\right)^g \widehat{\mathrm{var}}, \quad \text{especially} \quad g = 2 .$$

A detailed report will be available later.

In stratified random sampling with small sample size per stratum, the combined ratio estimator is often used. Let $W_h = N_h/N$ be the h^{th} stratum weight, $\bar{y}_h$, $\bar{x}_h$, $\bar{Y}_h$, $\bar{X}_h$ be the y- and x-sample and population means of the h^{th} stratum. The combined ratio estimator is

$$\hat{\bar{y}}_{Rc} = \frac{\bar{y}_{st}}{\bar{x}_{st}} \bar{X}, \quad \bar{y}_{st} = \sum_{1}^{L} W_h \bar{y}_h, \quad \bar{x}_{st} = \sum_{1}^{L} W_h \bar{x}_h .$$

Its approximate variance is

$$\sum_{1}^{L} \frac{1 - f_h}{n_h} \frac{1}{N_h - 1} \sum_{1}^{N_h} (y_{hi} - \bar{Y}_h - \frac{\bar{Y}}{\bar{X}} (x_{hi} - \bar{X}_h))^2 .$$

The following class of estimators

$$\left(\frac{\bar{X}}{\bar{x}_{st}}\right)^g \sum_{1}^{L} \frac{1-f_h}{n_h} \frac{1}{n_h-1} \sum_{1}^{n_h} (y_{hi} - \bar{y}_h - \frac{\bar{y}_{st}}{\bar{x}_{st}} (x_{hi} - \bar{x}_h))^2$$

has been considered by the first author (CFW). The case $g = 2$ is of special interest. The detailed results will be reported elsewhere.

Extensions in other situations are obvious.

REFERENCES

1. Brewer, K. R. W., Ratio estimation and finite populations: Some results deducible from the assumption of an underlying stochastic process, Australian Journal of Statistics 5 (1963), 93-105.
2. Cochran, W. G., Sampling Techniques, 3rd Edition, Wiley, New York, 1977.
3. Efron, B. and D. V. Hinkley, Assessing the accuracy of the maximum likelihood estimator: observed versus expected Fisher information, Biometrika 65 (1978), 457-487.
4. Fuller, W. A., A note on regression estimation for sample surveys, unpublished manuscript, 1977.
5. _______________, Comment on a paper by Royall and Cumberland, Journal of the American Statistical Association 76 (1981), 78-80.
6. Krewski, D. and R. P. Chakrabarty, On the stability of the jackknife variance estimator in ratio estimation, Journal of Statistical Planning and Inference (to appear).
7. Rao, J. N. K., Some small sample results in ratio and regression estimation, Journal of the Indian Statistical Association 6 (1968), 160-168.
8. _______________, Ratio and regression estimators, in New Developments in Survey Sampling (N. L. John and H. Smith, eds.), Wley, New York, 1969, 213-234.
9. _______________, Sampling designs involving unequal probabilities of selection and robust estimation of a finite population total, in Contributions to Survey Sampling and Applied Statistics (H. A. David, ed.), Academic Press, New York, 1978, 69-87.
10. _______________ and L. D. Beegle, A Monte Carlo study of some ratio estimators, Sankhya B 29 (1967), 47-56.
11. _______________ and R. A. Kuzik, Sampling errors in ratio estimation, Sankhya C 36 (1974), 43-58.
12. Rao, P. S. R. S. and J.N. K. Rao, Small sample results for ratio estimators, Biometrika 58 (1971), 625-630.

13. Royall, R. M., On finite population sampling theory under certain linear regression models, Biometrika 57 (1970), 377-387.

14. ______________ and W. G. Cumberland, Variance estimation in finite population sampling, Journal of the American Statistical Association 73 (1978), 351-358.

15. ______________ and ______________, An empirical study of prediction theory in finite population sampling: simple random samping and the ratio estimator, in Survey Sampling and Measurements (N.K. Namboodiri, ed.), Academic Press, New York, 1978, 293-309.

16. ______________ and ______________, An empirical study of the ratio estimator and estimators of its variance, Journal of the American Statistical Association 76 (1981), 66-77.

17. ______________ and K. R. Eberhardt, Variance estimates for the ratio estimator, Sankhya C 37 (1975), 43-52.

18. Wu, C. F., Estimation of variance of the ratio estimator, Biometrika 69 (1982), 183-189.

CW/LD/ed

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 2378	2. GOVT ACCESSION NO. AD-A116 144	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) ESTIMATION OF VARIANCE OF THE RATIO ESTIMATOR: AN EMPIRICAL STUDY		5. TYPE OF REPORT & PERIOD COVERED Summary Report - no specific reporting period
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Chien-Fu Wu and Lih-Yuan Deng		8. CONTRACT OR GRANT NUMBER(s) DAAG29-80-C-0041 MCS-7901846
9. PERFORMING ORGANIZATION NAME AND ADDRESS Mathematics Research Center, University of Wisconsin, 610 Walnut Street, Madison, Wisconsin 53706		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Work Unit Number 4 - Statistics and Probability
11. CONTROLLING OFFICE NAME AND ADDRESS (See Item 18 below)		12. REPORT DATE May 1982
		13. NUMBER OF PAGES 33
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

U. S. Army Research Office
P. O. Box 12211
Research Triangle Park
North Carolina 27709

National Science Foundation
Washington, DC 20550

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Ratio estimator, variance estimator, jackknife, regression estimator, ancillary statistic, conditional inference, superpopulation model, Monte Carlo

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Several estimators of variance of the ratio estimator in simple random sampling have been proposed in Cochran (1977), Fuller (1981), Royall and Cumberland (1978), Royall and Eberhardt (1975), Wu (1982). Their performances are compared on nine populations that reflect different features of natural populations encountered in practice. One criterion is the mean square error of the variance estimator as a point estimator of the variance of ratio; the other is the reliability of the associated t-interval. It turns out that the
(continued)

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

ABSTRACT (continued)

two criteria are not consistent. The apparent contradiction is resolved by a conditioning argument on an ancillary statistic, i.e., the reliability of the t-interval can be predicted by the closeness of the corresponding variance estimator to the conditional MSE of the ratio estimator on the ancillary statistic. Based on the empirical study, the jackknife estimator v_J and the estimator v_2 (and other asymptotically equivalent ones) are recommended. The good performance of these estimators is attributed to their ability in "capturing" the ancillary statistic.

FILMED

7-8